1.3

A POPULATION GROWS AT A RATE PROPORTIONAL TO THE EXISTING POPULATION

(EG. POPULATION IS GROWING AT 1.3%/YEAR)

$P(t)$ = POPULATION @ TIME t

$$\frac{dP}{dt} = k \cdot P \qquad k \in \mathbb{R}^+$$

$$\frac{dP}{dt} = kP$$ SINCE $P > 0$ AND $\frac{dP}{dt} > 0$, THEREFORE $k > 0$

SIMILAR TO RADIOACTIVE DECAY

$$\frac{dA}{dt} = -kA, \qquad k \in \mathbb{R}^+$$ SINCE $A > 0$ AND $\frac{dA}{dt} < 0$

NEWTON'S LAW OF HEATING & COOLING

THE TEMPERATURE OF AN OBJECT
CHANGES AT A RATE
PROPORTIONAL TO THE DIFFERENCE BETWEEN ITS TEMPERATURE
AND THE TEMPERATURE OF ITS SURROUNDINGS

(ASSUME TEMPERATURE OF ITS SURROUNDINGS IS FIXED)

$T(t)$ = TEMPERATURE OF OBJECT @ TIME t

$$\boxed{\frac{dT}{dt} = -k(T - T_s)} \qquad T_s \in \mathbb{R},\ k \in \mathbb{R}^+$$

IF $\frac{dT}{dt} < 0$, T IS DECR

TEMP OF OBJ IS DECR
TEMP OF SURR IS LOWER

$T > T_s$

$T - T_s > 0$

IF $\frac{dT}{dt} > 0$, T IS INCREASING

TEMP OF OBJ IS INCR
TEMP OF SURR IS HIGHER

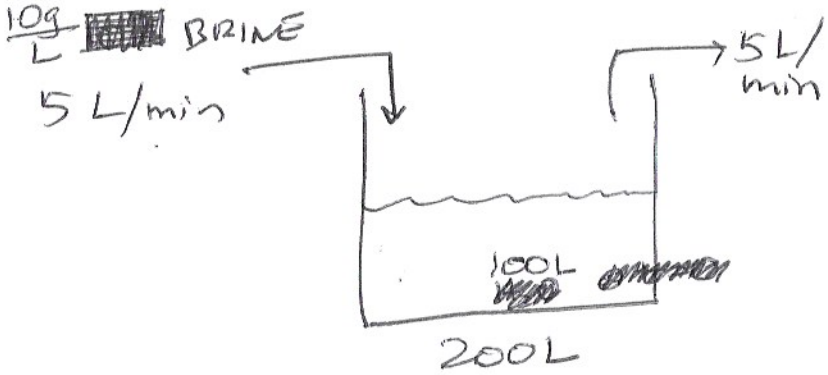
$T < T_s$

$T - T_s < 0$

so $k < 0$

OR $\frac{dT}{dt} = k(T_s - T)$

MIXING PROBLEM



A 200L TANK INITIALLY CONTAINS 100L OF WATER.

A 10g/L BRINE SOLUTION ENTERS THE TANK AT 5L/min (EACH 1L OF SOL'N CONTAINS 5g OF SALT)

AND THE WELLMIXED SOLUTION IN THE TANK LEAVES AT 5L/min.

FIND A DE FOR THE AMOUNT OF SALT IN THE TANK @ TIME t.

(ASSUME BRINE STARTS ENTERING AT t=0)

$A(t)$ = AMOUNT OF SALT IN TANK

$$\frac{dA}{dt} = \text{RATE SALT IN} - \text{RATE SALT OUT}$$

$$= \text{RATE BRINE IN} * \text{CONCENTRATION OF SALT IN BRINE} - \text{RATE SOL'N IN TANK OUT} * \text{CONCENTRATION OF SALT IN SOL'N IN TANK}$$

$$= \frac{5L}{min} * \frac{10g}{L} - \frac{5L}{min} * \frac{A\ g}{100L}$$

$$= \frac{50g}{min} - \frac{A\ g}{20min}$$

CONCENTRATION OF SALT IN TANK IS SAME EVERYWH

$$= \frac{\text{AMOUNT OF SALT IN TANK}}{\text{AMOUNT OF SOL'N IN TANK}}$$

$$\boxed{\frac{dA}{dt} = 50 - \frac{A}{20},\ A(0) = 0}$$

IVP

2.1 SLOPE FIELDS
or
DIRECTION FIELDS

CHAPTER 2
FIRST ORDER ODE

$$\frac{dy}{dx} = f(x,y)$$

$$G(x,y,y') = 0$$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

IF A GRAPH IS A SOLUTION OF THE IVP

$$\frac{dy}{dx} = f(x,y), \quad y(x_0) = y_0$$

$$y'(x_0) = \left.\frac{dy}{dx}\right|_{(x_0,y_0)} = f(x_0,y_0)$$

IN FACT, IF THE SOL'N PASSES THRU (a,b)
THE SLOPE OF THAT SOL'N AT (a,b) IS $f(a,b)$

LINK ON MY WEBSITE
FOR SLOPE FIELD GENERATOR